Calc AB Sem 1 Exam

Name: Block: Seat:

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Place your phone in your bag, and your bag in the front or back of the room

This exam has two parts. You may not use a calculator on Part 1, and must **turn in Part 1 before taking out your calculator for Part 2**, which allows for a calculator. It is fine to start with both parts so you may decide how much time you need to use your calculator on Part 2.

The following guidelines for this exam are the same as the AP exam:

i. Unless otherwise specified, answers (numeric or algebraic) need not be simplified. (Usually 5/10 or $\sqrt{12}$ is ok, but transcendental functions are not algebraic. If it is a transcendental function don't leave it as $\cos(\frac{\pi}{2})$; instead write 0. Instead of ln 1, write 0. Instead of e^0, write 1, etc.).

ii. On a Free response question, do remember to specify the units (if present), but don't waste time on arithmetic (arithmetic is $2 + 4 * 5$, not a formula like $a + 5b$; be sure to substitute actual values into a formula).

iii. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point. This means you should only round once, and as the last step. Store intermediate values ([STO >] [ALPHA] A) is a fast and accurate way to do this.

iv. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

v. Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the **clarity, correctness, and completeness** of your methods as well as your **answers**. Answers without supporting work will usually not receive credit (sometimes called a "bald" answer). Justifications require that you give mathematical (not calculator) reasons. You may need to show that the conditions for a theorem have been met before using the theorem to receive full credit.

	No Calculator	Calculator Active	
Multiple Choice	14*2 = 28 points	7*2 = 14 points	21 questions, 42 points (50%)
Free Response	2*14 = 28 points	1*14 = 14 points	3 questions, 42 points (50%)
	56 ($66.\overline{6}$%)	28 ($33.\overline{3}$%)	84 points
Recommended Time	60 min	30 min	90 min

Part 1 - (Calculators NOT Active) starts on the next Page

FR-I. Let f be defined as:

$$f(x) = \begin{cases} \sqrt{9-x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\frac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

(a) (3 points) Find the average rate of change of f on the interval $-3 \le x \le 4$

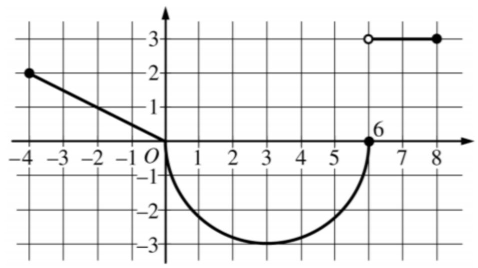
(b) (3 points) Write an equation for the line tangent to the graph of f at $x = 3$

Let f be defined as:

$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \leq x \leq 0 \\ -x + 3\cos\left(\frac{\pi x}{2}\right) & \text{for } 0 < x \leq 4 \end{cases}$$

(c) (5 points) Find the average value of $-3 \leq x \leq 4$ on the interval.
Hint: the sum of two integrals are involved here...

(d) (3 points) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \leq x \leq 4$? Justify your answer.



Graph of g

FR-II. The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above.

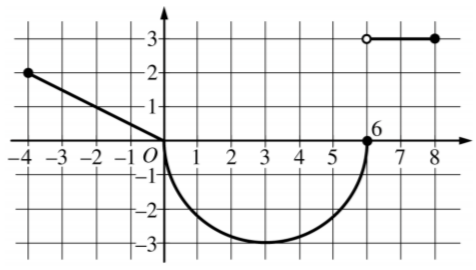
Let f be the function defined by

$$f(x) = 3x + \int_0^x g(t)\ dt.$$

(a) (3 points) Evaluate $\int_{-4}^{8} g(x)\ dx$

(b) (2 points) Find $\frac{d}{dx}\int_0^{x^2} g(t)dt$

(c) (2 points) Find $f'(x)$



Graph of g

The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above.

Let f be the function defined by

$$f(x) = 3x + \int_0^x g(t)\ dt.$$

(d) (3 points) Find $f(7)$ and $f'(7)$

(e) (2 points) Find the <u>value of x</u> in the closed interval $[-4, 3]$ at which f attains its maximum value. Justify your answer.

(f) (2 points) For each of $\lim_{x \to 0^-} g'(x)$ and $\lim_{x \to 0^+} g'(x)$, find the value or state that it does not exist.

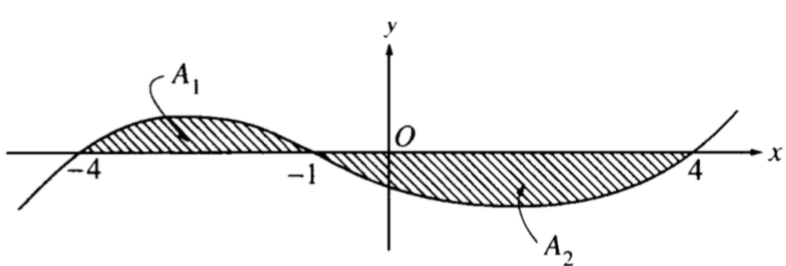
Multiple choice:
Justify your choice for both points

1. (2 points) Find the limit: $\lim_{x \to 2^-} \frac{1}{x-2}$
   (a) ∞
   (b) $-\infty$
   (c) 0
   (d) $-\frac{1}{4}$
   (e) None of these

2. (2 points) Find $\frac{dy}{dx}$ if $y^2 - 3xy + x^2 = 7$.
   (a) $\frac{2x}{3-2y}$
   (b) $\frac{2x}{y}$
   (c) $\frac{2x+y}{3x-2y}$
   (d) $\frac{3y-2x}{2y-3x}$
   (e) None of these

3. (2 points) Let $f'(x) = x^4 - x^2$ and let $f(x)$ have critical numbers $-1, 0,$ and 1. Use the Second Derivative Test to determine if any of the critical numbers gives a relative maximum.
   (a) -1
   (b) 0
   (c) 1
   (d) -1 and 1
   (e) None of these

4. (2 points) The graph of $y = f(x)$ is shown below. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2, $\int_{-4}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx =$

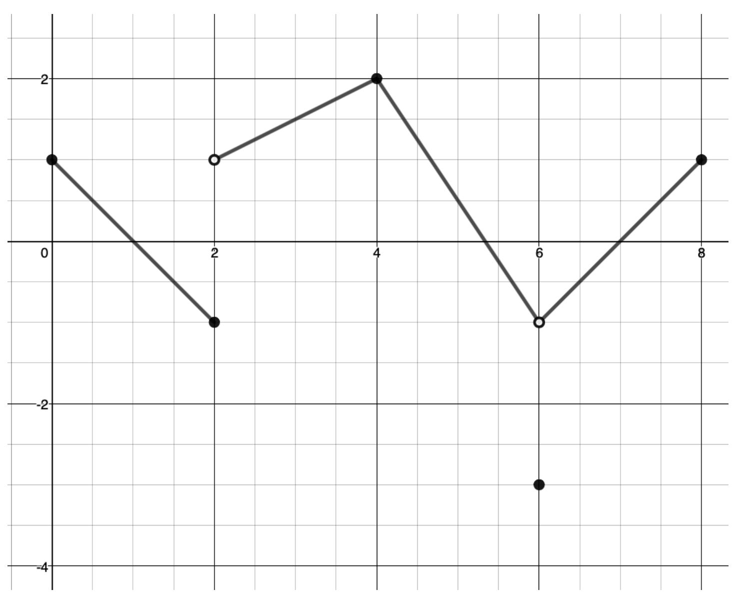


   (a) A_1
   (b) $A_1 - A_2$
   (c) $2A_1 - A_2$
   (d) $A_1 + A_2$
   (e) $A_1 + 2A_2$
   (f) None of these

5. (2 points) $\int_2^x (3t^2 - 1)\,dt =$
   (a) $x^3 - x - 6$
   (b) $x^3 - x$
   (c) $3x^2 - 12$
   (d) $3x^2 - 1$
   (e) $6x - 12$
   (f) None of these

6. (2 points) If $f(x) = 4x^{-2} + \frac{1}{4}x^2 + 4$, then $f'(2) =$
   (a) -62
   (b) -58
   (c) -3
   (d) 0
   (e) 1
   (f) None of these

7. (2 points) The figure below shows the graph of the function f. Which of the following statements are true?
   I. $\lim_{x\to 2^-} f(x) = f(2)$
   II. $\lim_{x\to 6^-} f(x) = \lim_{x\to 6^+} f(x)$
   III. $\lim_{x\to 6} f(x) = f(6)$



   (a) II only
   (b) III only
   (c) I and II only
   (d) II and III only
   (e) I, II, and III
   (f) None of these

8. Using the substitution $u = \sin(2x)$, $\int_{\pi/6}^{\pi/2} \sin^5(2x)\cos(2x)\ dx$ is equivalent to

   (a) $-2\int_{1/2}^{1} u^5\ du$

   (b) $\frac{1}{2}\int_{1/2}^{1} u^5\ du$

   (c) $\frac{1}{2}\int_{0}^{\sqrt{3}/2} u^5\ du$

   (d) $\frac{1}{2}\int_{\sqrt{3}/2}^{0} u^5\ du$

   (e) $2\int_{\sqrt{3}/2}^{0} u^5\ du$

   (f) None of these

9. (2 points) Let f be defined

$$f(x) = \begin{cases} \dfrac{x^2 - 7x + 10}{b(x-2)}, & x \neq 2 \\ b, & x = 2 \end{cases}$$

   For what value of b is f continuous at $x = 2$?

   (a) -3

   (b) $\sqrt{2}$

   (c) 3

   (d) 5

   (e) There is no such value of b

   (f) None of these

10. (2 points) Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)\ dx$

    (a) 20

    (b) 40

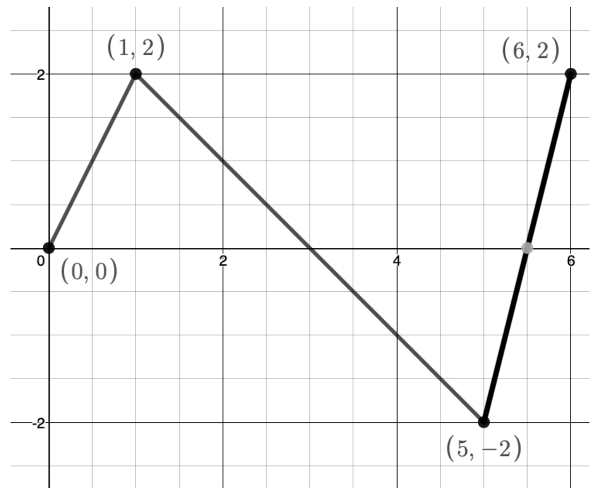
    (c) 60

    (d) 80

    (e) 120

    (f) None of these

11. (2 points) For $0 \le x \le 6$, the graph of f', the derivative of f, is piecewise linear as shown below. If $f(0) = 1$, what is the maximum value of f on $[0, 6]$?



   (a) 1
   (b) 1.5
   (c) 2
   (d) 4
   (e) 6
   (f) None of these

12. (2 points) Functions w, x, and y are differentiable with respect to time and are related by the equation $w = x^2y$. If x is decreasing at a constant rate of 1 unit per minute and y is increasing at a constant rate of 4 units per minute, at what rate is w changing with respect to time when $x = 6$ and $y = 20$?
   (a) -340
   (b) -240
   (c) -96
   (d) 276
   (e) 384

13. (2 points) Let f be the function defined by $2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both decreasing and concave up?
   (a) $(-\infty, -1)$
   (b) $\left(-1, \frac{1}{2}\right)$
   (c) $(-1, 2)$
   (d) $\left(\frac{1}{2}, 2\right)$
   (e) $(2, \infty)$
   (f) None of these

14. (2 points) What is the slope of the line tangent to the curve $2y^2 - 3x^2 = 6 - 2xy$ at the point $(2, 3)$
   (a) 0
   (b) $\frac{3}{8}$
   (c) $\frac{7}{9}$
   (d) $\frac{6}{7}$
   (e) $\frac{1}{11}$

Turn in Part 1 before using your calculator

Part 2 Calculators Active. (Permitted, though not always necessary)
Give final answers to three decimal places, either rounded or truncated

101. (2 points) The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

   (a) 2.545 meters
   (b) 10.263 meters
   (c) 34.125 meters
   (d) 54.889 meters
   (e) 89.005 meters
   (f) None of these

102. (2 points) If $\sin\left(\dfrac{1}{x^2+1}\right)$ is an antiderivative for $f(x)$, then $\displaystyle\int_1^2 f(x)\,dx =$

   (a) -0.281
   (b) -0.102
   (c) 0.102
   (d) 0.260
   (e) 0.282
   (f) 0.3
   (g) None of these

103. The table below shows selected values of a function f. The function is twice differentiable with $f''(x) > 0$. Which of the following could be the value of $f'(3)$?

x	1	3	5
$f(x)$	2.4	3.6	5.4

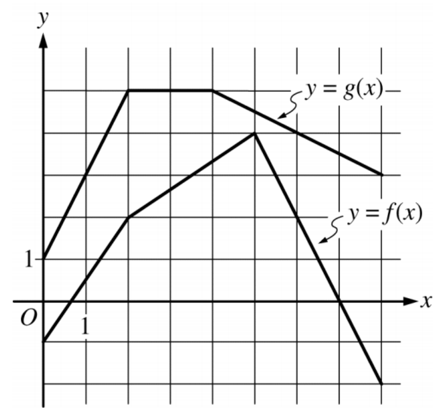
   (a) 0.6
   (b) 0.7
   (c) 0.9
   (d) 1.2
   (e) 1.5
   (f) None of these

104. (2 points) A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

   Hint: Use [MATH] [`8.nDeriv`] on your calculator

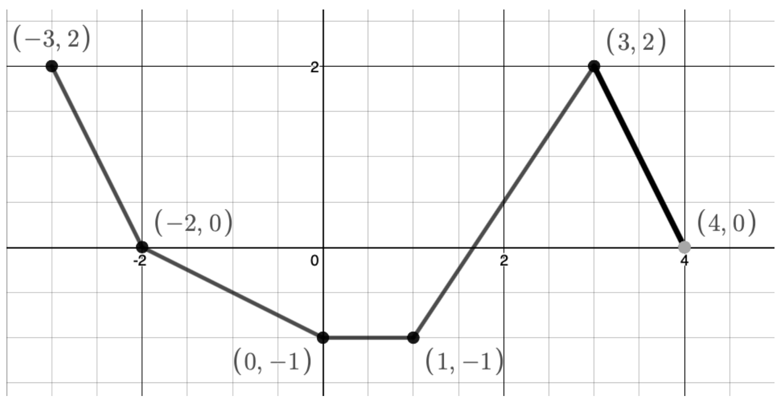
   (a) 1.633
   (b) -2.016
   (c) 1.814
   (d) 2.978
   (e) None of these

105. (2 points) The graphs of f and g are shown below.
If $h(x) = f(x) \cdot g(h)$, then $h'(6) =$



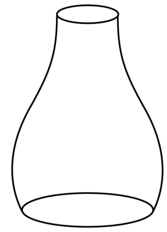
- (a) -9
- (b) -7
- (c) 1
- (d) 7
- (e) 9
- (f) None of these

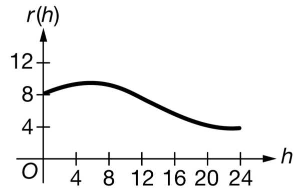
Given the graph of the function f below, and let g be the function $g(x) = \int_{-2}^{x} f(t)\, dt$



106. (2 points) g is increasing on the approximate interval
- (a) $(-2, 1.7)$ only
- (b) $(1, 3)$ only
- (c) $(1.7, 4)$ only
- (d) both $(-3, -2)$ and $(1.7, 4)$
- (e) $(0, 1)$ only

107. The position equation for the movement of a particle is given by $s = (t^2 - 1)^3$ when s is measured in feet and t is measured in seconds. Find the acceleration at two seconds.
- (a) 18 feet/sec^2
- (b) 90 feet/sec^2
- (c) 288 feet/sec^2
- (d) 342 feet/sec^2
- (e) None of these



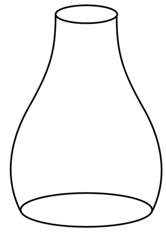


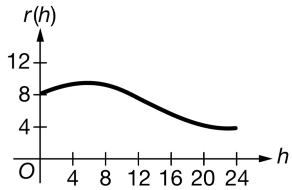
FR-III. A 24-centimeter tall vase has cross sections parallel to the base that are circles. Each circular cross section h centimeters above the base has a radius of $r(h) = 8 - 0.1h + 2\sin(0.3h^{0.9})$ centimeters for $0 \le h \le 24$. A sketch of the vase and the graph of r are shown above.

(a) (3 points) Find the area of the region between the graph of r and the h-axis from $h = 0$ to $h = 24$. (*points awarded for integral and answer with units*)

(b) (2 points) Evaluate $\int_4^{20} r'(h)dh$. Using correct units, interpret the meaning of $\int_4^{20} r'(h)dh$ in the context of the problem.

(c) (2 points) Evaluate $\frac{1}{24}\int_0^{24} r(h)dh$. Using correct units, interpret the meaning of $\frac{1}{24}\int_0^{24} r(h)dh$. in the context of the problem.





A 24-centimeter tall vase has cross sections parallel to the base that are circles. Each circular cross section h centimeters above the base has a radius of $r(h) = 8 - 0.1h + 2\sin(0.3h^{0.9})$ centimeters for $0 \le h \le 24$. A sketch of the vase and the graph of r are shown above.

(d) (4 points) Water is poured into the vase. At the instant when the depth of the water in the vase is 10 centimeters, the depth of the water is increasing at a rate of 0.6 centimeter per second. At that instant, what is the rate of change of the circumference of the surface of the water, in centimeters per second? (Recall the circumference of a circle is $C = 2\pi r$)

Hint: find $\frac{dC}{dt}$ using chain rule and MATH `8.nDeriv` to find $\frac{dr}{dh}|_{h=10}$

(e) (3 points) The area of a circle is πr^2 so the volume of the vase is $\displaystyle\int_0^{24} \pi(r(h))^2\, dh$. Find the volume of the vase when the height of the water is 12 centimeters.

(*points awarded for integral and answer with units*)